

SOME QUADRATIC CORRECT EXTENSIONS OF MINIMAL OPERATORS IN BANACH SPACES

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Abstract. Let A_0 be a minimal operator from a complex Banach space X into X with finite defect, $\text{def } A_0 = m$, and \widehat{A} is a linear correct extension of A_0 . Let $E_c(A_0, \widehat{A})$ (resp. $E_c(A_0^2, \widehat{A}^2)$) denote the set of all correct extensions B of A_0 with domain $D(B) = D(\widehat{A})$ (resp. B_1 of A_0^2 with $D(B_1) = D(\widehat{A}^2)$) and let $E_c^m(A_0, \widehat{A})$ (resp. $E_c^{m+k}(A_0^2, \widehat{A}^2)$, $k \leq m$, $k, m \in \mathbf{N}$) denote the subset of $E_c(A_0, \widehat{A})$ (resp. $E_c(A_0^2, \widehat{A}^2)$) consisting of all $B \in E_c(A_0, \widehat{A})$ (resp. $E_c(A_0^2, \widehat{A}^2)$) such that $\dim R(B - \widehat{A}) = m$ (resp. $\dim R(B_1 - \widehat{A}^2) = m + k$). In this paper:

1. we characterize the set of all operators $B_1 \in E_c^{m+k}(A_0^2, \widehat{A}^2)$ with the help of \widehat{A} and some vectors S and G and give the solution of the problem $B_1 x = f$,
2. we describe the subset $E_c^{2m}(A_0^2, \widehat{A}^2)$ of all operators $B_2 \in E_c^{2m}(A_0^2, \widehat{A}^2)$ such that $B_2 = B^2$, where B is an operator of $E_c^m(A_0, \widehat{A})$ corresponding to B_2 ,
3. we give the solution of problems $B_2 x = f$.

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