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FINITE INTERTWININGS AND SUBSCALARITY

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Abstract. Quasinilpotent equivalence does not preserve subscalarity. However, if we replace quasinilpotent equivalence by "finite intertwining by the identity operator", then subscalarity is preserved (in one direction). We shall prove that if A, B and N are Banach space operators such that $\triangle^n_{AB}(I) = \triangle_{AB}(\triangle^{n-1}_{AB}(I)) = \sum_{i=0}^n (-1)^i \binom{n}{i} A^{n-i}B^i = 0$ for some positive integer n, and if N is an algebraic operator which commutes with B, then A is subscalar implies $B+N$ is subscalar. Applications to classes of Hilbert space operators, and the elementary operators $L_A - R_B$ and $L_AR_B - 1$ for certain choices of subscalar operators A and B^*, are considered.

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