

ON COMMUTATORS IN MATRIX THEORY

GEOFFREY R. GOODSON

Abstract. We investigate intertwining relations arising from commutators such as $AB - BA = D$ when $AD = DA$, and $AB - BA^T = D$ when $AD = DA^T$, where A, B and D are n -by- n matrices. Depending on the properties of A , such equations often force D to be zero or at least nilpotent, and it is the properties of D that we investigate. We briefly discuss the situation when $AB + BA = D$, $AD = DA^T$ for A normal.

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