Operators and Matrices
Volume 4, Number 3 (2010), 405–415

FURUTA INEQUALITY AND q–HYPONORMAL OPERATORS

JIANGTAO YUAN

Abstract. This paper is to consider Furuta type inequalities and q-hyponormal operators. It is shown that the complete form and original form of Furuta inequality are equivalent to each other. Forward, we prove that the complete form and original form of Furuta inequality are equivalent to the order relations among Aluthge transforms on q-hyponormal operators. Lastly, a simplified and short proof of the order structure on powers of q-hyponormal operators is shown.

Mathematics subject classification (2010): 47A63, 47B20.
Keywords and phrases: Loewner-Heinz inequality, Furuta inequality, q-hyponormal operator, Aluthge transform.

REFERENCES

[1] A. ALUTHGE, *On p-hyponormal operators for $0 < p < 1$*, Integral Equations Operator Theory, **13** (1990), 307–315.
[2] A. ALUTHGE AND D. WANG, *Powers of p-hyponormal operators*, J. Inequal. Appl., **3** (1999), 279–284.
[3] A. ALUTHGE AND D. XIA, *A trace estimate of $(T^*T)^p - (TT^*)^p$*, Integral Equations Operator Theory, **12** (1989), 300–303.
[4] J. ANTEZANA, E. PUJALS AND D. STOJANOFF, *Convergence of iterated Aluthge transform sequence for diagonalizable matrices*, Adv. Math., **216** (2007), 255–278.
[5] M. CHŌ, I. B. JUNG AND W. Y. LEE, *On Aluthge transforms of p-hyponormal operators*, Integral Equations Operator Theory, **53** (2005), 321–329.
[6] R. CURTO, P. MUHLY AND D. XIA, *A trace estimate of the p-hyponormal operators*, Integral Equations Operator Theory, **6** (1983), 507–514.
[7] M. FUJII AND Y. NAKATSU, *On subclasses of hyponormal operators*, Proc. Japan Acad. Ser. A Math. Sci., **51** (1975), 243–246.
[8] T. FURUTA, *$A \geqslant B \geqslant 0$ assures $(B^rA^pB^r)^{1/q} \geqslant B^{(p+2r)/q}$ for $r \geqslant 0$, $p \geqslant 0$, $q \geqslant 1$ with $(1+2r)q \geqslant p+2r$*, Proc. Amer. Math. Soc., **101** (1987), 85–88.
[9] T. FURUTA, *Invitation to Linear Operators*, Taylor & Francis, London, 2001.
[10] T. FURUTA AND M. YANAGIDA, *On powers of p-hyponormal and $\log$-hyponormal operators*, J. Inequal. Appl., **5** (2000), 367–380.
[11] S. R. GARCIA, *Aluthge transforms of complex symmetric operators*, Integral Equations Operator Theory, **60** (2008), 357–367.
[12] P. R. HALMOS, *Normal dilations and extensions of operators,* Summa Bras. Math., **2** (1950), 125–134.
[13] P. R. HALMOS, *A Hilbert space problem book*, 2nd ed. Springer-Verlag, New York, 1982.
[14] T. HURUYA, *A note on p-hyponormal operators*, Proc. Amer. Math. Soc., **125** (1997), 3617–3624.
[15] M. ITO, *Generalizations of the results on powers of p-hyponormal operators*, J. Inequal. Appl., **6** (2000), 1–15
[16] M. ITO AND T. YAMAZAKI, *Relations between two inequalities $(B^{\frac{r}{2}}A^pB^{\frac{r}{2}})^{\frac{r}{p+r}} \geqslant B^r$ and $(A^{\frac{p}{2}}B^rA^{\frac{p}{2}})^{\frac{p}{p+r}} \leqslant A^p$ and its applications*, Integral Equations Operator Theory, **44** (2002), 442–450.
[17] M. ITO, T. YAMAZAKI AND M. YANAGIDA, *Genaralications of results on relations between Furuta-type inequalities*, Acta Sci. Math. (Szeged), **69** (2003), 853–862.

Paper OaM-04-21

[18] G. JI, Y. PANG AND Z. LI, *On the range of the Aluthge transform*, Integral Equations Operator Theory, **57** (2007), 209–215.
[19] I. JUNG, E. KO AND C. PEARCY, *Aluthge transform of operators*, Integral Equations Operator Theory, **37** (2000), 437–448.
[20] M. MARTIN AND M. PUTINAR, *Lectures on Hyponormal Operators*, Birkhauser Verlag, Boston, 1989.
[21] K. TANAHASHI, *Best possibility of Furuta inequality*, Proc. Amer. Math. Soc., **124** (1996), 141–146.
[22] P. Y. WU, *Numerical range of Aluthge transform of operator*, Linear Algebra Appl., **357** (2002), 295–298.
[23] D. XIA, *On the nonnormal operators-semihyponormal operators*, Sci. China Ser. A, **23** (1980), 700–713.
[24] D. XIA, *Spectral Theory of Hyponormal Operators*, BirkhauserVerlag, Boston, 1983.
[25] T. YAMAZAKI, *Extensions of the results on p-hyponormal and* log*-hyponormal operators by Aluthge and Wang*, SUT J. Math., **35** (1999), 139–148.
[26] M. YANAGIDA, *Some applications of Tanahashi's result on the best possibility of Furuta inequality*, Math. Inequal. Appl., **2** (1999), 297–305.
[27] C. YANG AND J. YUAN, *Extensions of the results on powers of p-hyponormal and* log*-hyponormal operators*, J. Inequal. Appl., **2006** (2006), Article ID 36919, 1–14.
[28] T. YOSHINO, *The p-hyponormality of the Aluthge transform*, Interdiscip. Inform. Sci., **3** (1997), 91–93.
[29] J. YUAN AND Z. GAO, *Structure on powers of p-hyponormal and* log*-hyponormal operators*, Integral Equations Operator Theory, **59** (2007), 437–448.
[30] J. YUAN AND Z. GAO, *The operator equation $K^p = H^{\frac{\delta}{2}} T^{\frac{1}{2}} (T^{\frac{1}{2}} H^{\delta+r} T^{\frac{1}{2}})^{\frac{p-\delta}{\delta+r}} T^{\frac{1}{2}} H^{\frac{\delta}{2}}$ and its applications*, J. Math. Anal. Appl., **341** (2008), 870–875.
[31] J. YUAN AND Z. GAO, *Complete form of Furuta inequality*, Proc. Amer. Math. Soc., **136** (8) (2008), 2859–2867.