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CHARACTERIZATION OF ξ–LIE MULTIPLICATIVE ISOMORPHISMS

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Abstract. Let $\mathscr{A}$ and $\mathscr{A}'$ be two algebras over a field $\mathbb{F}$ and $\xi \in \mathbb{F}$ a scalar. A map $\Phi : \mathscr{A} \to \mathscr{A}'$ is called a ξ-Lie multiplicative isomorphism if Φ is bijective and satisfies $\Phi(AB - \xi BA) = \Phi(A)\Phi(B) - \xi\Phi(B)\Phi(A)$ for all $A, B \in \mathscr{A}$. The additivity of ξ-Lie multiplicative isomorphisms on prime algebras is discussed. A characterization of ξ-Lie multiplicative isomorphisms between matrix algebras over a field of characteristic not 2 and a characterization of ξ-Lie multiplicative isomorphisms between infinite dimensional Banach space standard operator algebras are obtained.

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REFERENCES

[1] R. AN, J. HOU, *Additivity of Jordan multiplicative maps on Jordan operator algebras*, Taiwanese J. Math., **10** (2006), 45–64.
[2] Z. BAI, S. DU, J. HOU, *Multiplicative Lie isomorphisms between prime rings*, Communication in Algebras, **36** (2008), 1626–1633.
[3] J. A. BROOKE, P. BUSCH, B. PEARSON, *Commutativity up to a factor of bounded operators in complex Hilbert spaces*, R. Soc. Lond. Proc. Ser. A Math Phys. Eng. Sci., A **458** (2002), 109–118.
[4] C.G. CAO, X. ZHANG, *Additive surjections preserving rank one and applications*, Georgian Math. J., **11** (2004), 209–217.
[5] J. CUI, J. HOU, *A characterization of automorphisms on nest algebras*, Chinese Ann. Math., **23** (2002), 521–530.
[6] J. CUI, J. HOU, CHOONKIL PARK, *Additive maps preserving commutativity up to a factor*, Chinese Ann. Math., **5** (2008), 583–590.
[7] O. HOLTZ, V. MEHRMANN, AND H. SCHNEIDER, *Potter, Wielandt, and Drazin on the matrix equation $AB = \omega BA$: New answers to old questions*, Amer. Math. Monthly, **111** (2004), 655–667.
[8] C. KASSEL, *Quantum groups*, Springer-Verlag, New York, 1995.
[9] F. LU, *Multiplicative mappings of operator algebras*, Lin. Alg. Appl., **347** (2002), 283–291.
[10] W. S. MARTINDALE III, *When are multiplicative mappings additive?*, Proc. Amer. Math. Soc., **21** (1969), 695–698.
[11] P. ŠEMRL, *Non-linear commutativity preserving maps*, Acta Sci. Math. (Szeged), **71** (2005), 781–819.
[12] C. R. PUTNAM, *Commutation properties of Hilbert space operators and related topics*, Springer-Verlag, New York, 1967.
[13] Z. WAN, *Geometry of Matrices*, World Scientific, Singapore, New Jersey, Longdon, Hongkong, 1996.
[14] J. YANG, H. DU, *A note on commutativity up to a factor of bounded operators*, Proc. Amer. Math. soc., **132** (2004), 1713–1720.
[15] L. ZHAO, J. HOU, *Jordan zero-product preserving additive maps on operator algebras*, J. Math. Anal. Appl., **314** (2006), 689–700.

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