

## COLUMN SUMS AND THE CONDITIONING OF THE STATIONARY DISTRIBUTION FOR A STOCHASTIC MATRIX

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*Abstract.* For an irreducible stochastic matrix  $T$ , we consider a certain condition number  $\kappa(T)$ , which measures the sensitivity of the stationary distribution vector to perturbations in  $T$ , and study the extent to which the column sum vector for  $T$  provides information on  $\kappa(T)$ . Specifically, if  $c^T$  is the column sum vector for some stochastic matrix of order  $n$ , we define the set  $\mathcal{S}(c) = \{A|A \text{ is an } n \times n \text{ stochastic matrix with column sum vector } c^T\}$ . We then characterise those vectors  $c^T$  such that  $\kappa(T)$  is bounded as  $T$  ranges over the irreducible matrices in  $\mathcal{S}(c)$ ; for those column sum vectors  $c^T$  for which  $\kappa$  is bounded, we give an upper bound on  $\kappa$  in terms of the entries in  $c^T$ , and characterise the equality case.

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