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MULTIPLIERS OF MULTIDIMENSIONAL FOURIER ALGEBRAS

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Abstract. Let G be a locally compact σ-compact group. Motivated by an earlier notion for discrete groups due to Effros and Ruan, we introduce the multidimensional Fourier algebra $A^n(G)$ of G. We characterise the completely bounded multidimensional multipliers associated with $A^n(G)$ in several equivalent ways. In particular, we establish a completely isometric embedding of the space of all n-dimensional completely bounded multipliers into the space of all Schur multipliers on G^{n+1} with respect to the (left) Haar measure. We show that in the case G is amenable the space of completely bounded multidimensional multipliers coincides with the multidimensional Fourier-Stieltjes algebra of G introduced by Ylinen. We extend some well-known results for abelian groups to the multidimensional setting.

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