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LINEAR MAPS PRESERVING THE MINIMUM AND SURJECTIVITY MODULI OF OPERATORS

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Abstract. Let $\mathscr{B}(H)$ be the algebra of all bounded linear operators on a complex Hilbert space H, and denote by $\mathrm{m(T)}$ and $\mathrm{q(T)}$ respectively the minimum modulus and the surjectivity modulus for every $T \in \mathscr{B}(H)$. In this paper, we prove that if ϕ is a surjective unital linear map on $\mathscr{B}(H)$, then $\mathrm{m}(\phi(\mathrm{T})) = \mathrm{m(T)}$ for every $T \in \mathscr{B}(H)$ if and only if $\mathrm{q}(\phi(\mathrm{T})) = \mathrm{q(T)}$ for every $T \in \mathscr{B}(H)$ if and only if there exists an unitary operator $U \in \mathscr{B}(H)$ such that $\phi(T) = UTU^*$ for all $T \in \mathscr{B}(H)$.

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