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NON–COMMUTATIVE INDEPENDENCE OF ALGEBRAS AND APPLICATIONS TO PROBABILITY

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Abstract. We present the notions of independence, which appear in non-commutative probability. The basic ones are free, boolean and monotonic independences, formulated for families of algebras indexed by totally ordered set. A generalization of the latter two is the bm-independence, defined for partially ordered index sets. For each independence there is an analogue of the classical central limit theorem. In the case of bm-independence this depends also on the index set. Examples of such partially ordered index sets are discrete lattices in symmetric positive cones.

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REFERENCES

[1] D. AVITZOUR, *Free Product of C^*-Algebras*, Trans. Amer. Math. Soc., **271**, 2 (1982), 423–435.
[2] M. BOŻEJKO, *Positive definite functions on the free group and the non-commutative Riesz product*, Boll. Un. Mat. Ital., **6**, 5A (1986), 13–21.
[3] J. FARAUT, A. KORANYI, *Analysis on Symmetric Cones*, Oxford Univ. Press 1994.
[4] U. FRANZ, *Monotone independence is associative*, Infin. Dimens. Anal. Quantum Probab. Relat. Top., **4**, 3 (2001), 401–407.
[5] Y. G. LU, *An interacting Fock space and the arcsine law*, Probab. Math. Stat., **17** (1997), 149–166.
[6] N. MURAKI, *A new example of non-commutative "de Moivre-Laplace theorem"*, Prob.Th. Math. Stat., Proceedings 7th Japan-Russia Symposium, Tokyo 1995, ed. Watanabe, S. et al., World Scientific (1996), 353–362.
[7] N. MURAKI, *Monotonic independence, monotonic central limit theorem and monotonic law of small numbers*, Infin. Dimens. Anal. Quantum Probab. Relat. Top., **4**, 1 (2001), 39–58.
[8] D. VOICULESCU, *Symmetries of some reduced free product C^*-algebras*, in Operator algebras and their connections with topology and ergodic theory (Buşteni, 1983), 556–588, Lecture Notes in Math., 1132, Springer, Berlin, 1985.
[9] J. WYSOCZAŃSKI, *Monotonic independence on weakly monotone Fock space*, Inf. Dim. Anal. Quantum Prob. Rel. Top., **8**, 2 (2005), 259–275.
[10] J. WYSOCZAŃSKI, *Monotonic independence associated with partially ordered sets*, Infin. Dimens. Anal. Quantum Probab. Relat. Top., **10**, 1 (2007), 17–41.
[11] J. WYSOCZAŃSKI, *bm-Central Limit Theorems for Positive Definite Real Symmetric Matrices*, Infin. Dimens. Anal. Quantum Probab. Relat. Top., **11**, 1 (2008), 1–19.

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