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ON L^2–EIGENFUNCTIONS OF TWISTED LAPLACIAN ON CURVED SURFACES AND SUGGESTED ORTHOGONAL POLYNOMIALS

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Abstract. We show in a unified manner that the factorization method describes completely the L^2-eigenspaces associated to the discrete part of the spectrum of the twisted Laplacian on constant curvature Riemann surfaces. Subclasses of two variable orthogonal polynomials are then derived and arise by successive derivations of elementary complex valued functions depending on the geometry of the surface.

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