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NON METRIZABILITY OF SOME TOPOLOGIZABLE ALGEBRAS

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Abstract. We compare topologization of certain algebras as topological algebras as well as metrizable semitopological algebras. The main aim of the paper is to prove that the algebra of continuous functions on a non Lindelöf paracompact space is not topologizable as a metrizable semitopological algebra. This gives us further examples of locally convex topologizable algebras that are not topologizable as metrizable semitopological algebras. On the other hand we prove that the free algebra generated by $\aleph$ variables has at last $\max\{2^{\aleph_0}, \aleph\}$ topologies of metrizable semitopological locally convex algebras. We provide also, for all $p \in (0,1]$, an example of locally bounded topological algebra which is not topologizable as a metrizable semitopological locally p-convex algebra. This is an investigation in the direction of some results of J. Esterle, V. Müller, W. Żelazko and the author in previous papers.

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