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BOUNDARY INTEGRAL METHODS FOR WEDGE DIFFRACTION PROBLEMS: THE ANGLE $2\pi/n$, DIRICHLET AND NEUMANN CONDITIONS

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Abstract. In this paper we use analytical methods for boundary integral operators (more precisely, pseudodifferential operators) together with symmetry arguments in order to treat harmonic wave diffraction problems in which the field does not depend on the third variable and the wave incidence is perpendicular. These problems are formulated as two-dimensional, mixed elliptic boundary value problems in a non-rectangular wedge.

We solve explicitly a number of reference problems for the Helmholtz equation regarding particular wedge angles, boundary conditions, and space settings, which can be modified and generalized in various ways. The solution of these problems in Sobolev spaces was open for some fifty years.

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