MULTIPlicITIES, BOUNDARY POINTS, AND JOINT NUMERICAL RANGES

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Abstract. The multiplicity of a point in the joint numerical range $W(A_1, A_2, A_3) \subseteq \mathbb{R}^3$ is studied for $n \times n$ Hermitian matrices $A_1, A_2, A_3$. The relative interior points of $W(A_1, A_2, A_3)$ have multiplicity greater than or equal to $n - 2$. The lower bound $n - 2$ is best possible. Extreme points and sharp points are studied. Similar study is given to the convex set $V(A) := \{x^T A x : x \in \mathbb{R}^n, x^T x = 1\} \subseteq \mathbb{C}$, where $A \in \mathbb{C}^{n \times n}$ is symmetric. Examples are given.


Keywords and phrases: Joint numerical range, multiplicity, extreme point, sharp point, boundary point.

REFERENCES