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INTEGRAL ESTIMATES FOR THE FAMILY OF B–OPERATORS

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Abstract. Let $\mathscr{P}_n$ be the class of polynomials of degree at most n. In 1969, Rahman introduced a class $\mathscr{B}_n$ of operators B that map $\mathscr{P}_n$ into itself and proved that

$$\|B[P(R\,\cdot)]\|_\infty \leqslant |B[E_n(R\,\cdot)]|\|P\|_\infty, \quad R \geqslant 1,$$

for every $B \in \mathscr{B}_n$, where $E_n(z) := z^n$.

In this paper, we show that this inequality holds analogously for the norm $\|\cdot\|_q$ with $q \geqslant 1$ and for some of its refinements as well.

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