

A NOTE ON THE FREDHOLM PROPERTIES OF TOEPLITZ OPERATORS ON WEIGHTED BERGMAN SPACES WITH MATRIX-VALUED SYMBOLS

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Abstract. We characterize the essential spectra of Toeplitz operators T_a on weighted Bergman spaces with matrix-valued symbols; in particular we deal with two classes of symbols, the Douglas algebra $C+H^\infty$ and the Zhu class $\mathcal{Q} := L^\infty \cap VMO_\partial$. In addition, for symbols in $C+H^\infty$, we derive a formula for the index of T_a in terms of its symbol a in the scalar-valued case, while in the matrix-valued case we indicate that the standard reduction to the scalar-valued case fails to work analogously to the Hardy space case.

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