

## AMBARZUMYAN-TYPE THEOREMS ON STAR GRAPHS

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*Abstract.* The so-called Ambarzumyan theorem states that if the Neumann eigenvalues of the Sturm-Liouville operator  $-\frac{d^2}{dx^2} + q$  with an integrable real-valued potential  $q$  on  $[0, \pi]$  are  $\{n^2 : n \geq 0\}$ , then  $q = 0$  for almost all  $x \in [0, \pi]$ . In this work, the classical Ambarzumyan theorem is extended to star graphs with Dirac operators on its edges. We prove that if the spectrum of Dirac operator on star graphs coincides with for the unperturbed case, then the potential is identically zero.

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