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PROJECTIONS AND IDEMPOTENTS WITH FIXED DIAGONAL AND THE HOMOTOPY PROBLEM FOR UNIT TIGHT FRAMES

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Abstract. We investigate the topological and metric structure of the set of idempotent operators and projections which have prescribed diagonal entries with respect to a fixed orthonormal basis of a Hilbert space. As an application, we settle some cases of conjectures of Larson, Dykema, and Strawn on the connectedness of the set of unit-norm tight frames.

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