Operators and Matrices
Volume 5, Number 1 (2011), 157–171

STRONGLY SPLITTING WEIGHTED SHIFT OPERATORS ON BANACH SPACES AND UNICELLULARITY

M. T. KARAEV AND M. GÜRDAL

Abstract. We introduce the notion of strong splitting operator on a separable Banach space, and prove a structure theorem for this operator. We consider the weighted shift operator T, $Te_n = \lambda_n e_{n+1}$, $n \geqslant 0$, acting in the Banach space X with basis $\{e_n\}_{n\geqslant 0}$. We give some sufficient conditions for X and for the weight sequence $\{\lambda_n\}_{n\geqslant 0}$ under which the operator is unicellular, that is, every nontrivial invariant subspace E of T has the form $E = X_i := \mathrm{Span}\,\{e_k : k \geqslant i\}$ for some $i \geqslant 1$; and prove that the restricted operators $T|X_i$ $(i \geqslant 1)$ are strong splitting. Moreover, we describe in terms of so-called discrete Duhamel operator and diagonal operator all extended eigenvectors of the operators $T|X_i$ $(i \geqslant 1)$.

Mathematics subject classification (2010): Primary 47B38, 47A15; Secondary 46B15.
Keywords and phrases: Banach space, weighted shift operator, strong splitting operator, invariant subspace, Duhamel product.

REFERENCES

[1] N. K. NIKOLSKI, *Treatise on the shift operator*, vol. 273, Springer, Berlin, 1986.
[2] M. T. KARAEV, *On extended eigenvalues and extended eigenvectors of some operator classes*, Proc. Amer. Math. Soc., **134** (2006), 2383–2392.
[3] N. K. NIKOLSKI, *Basisness and unicellularity of weighted shift operators*, Izvestiya Akad. Nauk SSSR, Ser. Mat., **32** (1968), 1123–1137 (in Russian).
[4] M. T. KARAEV, *On some applications of the ordinary and extended Duhamel products*, Sibirskii Matem. Zhurnal, **46**, 3(2005), 553–566 (in Russian).
[5] M. GÜRDAL, *Description of extended eigenvalues and extended eigenvectors of integration operator on the Wiener algebra*, Expo. Math., **27**, 2 (2009), 153–160.
[6] M. T. KARAEV, *Addition of spectral multiplicities and invariant subspaces*, Ph. D. Thesis, Baku, 1991 (in Russian).
[7] M. T. KARAEV, *An addition theorem on multiplicities of spectrum*, Trudy IMM AN Azerb., **8**, 16 (1998), 123–128 (in Russian).
[8] A. SHIELDS, *Weighted shift operators and analytic function theory. Topics in operator theory*, 128, Math. Surveys, No:13, Amer. Math. Soc., Providence, R.I., 1974, pp. 49.
[9] V. M. KADETS, *Bases with individual brackets and bases with individual permutations* (Russian), Teor. Funkt. i Funktsional Anal. i Prilozhen, **49** (1988) , 43–51; Translation in J. Soviet. Math., **49** (1990), 1064–1069.
[10] N. K. NIKOLSKI, *Selected problems of weighted approximation and spectral analysis*, Translated from Proceedings of the Steklov Institute of Mathematics, **120** (1974), 1–270; American Math. Soc., Providence, R.I., 1976, 276pp.
[11] L. V. KANTOROVICH AND G. P. AKILOV, *Functional Analysis*, Nauka, Moscow, 1977.
[12] A. BISWAS, A. LAMBERT AND S. PETROVIC, *Extended eigenvalues and the Volterra operator*, Glasgow Math. J., **44** (2002), 521–534.
[13] A. BISWAS, A. LAMBERT AND S. PETROVIC, *On extended eigenvectors for operators*, Preprint.
[14] A. LAMBERT, *Hyperinvariant subspaces and extended eigenvalues*, New York J. Math., **10** (2004), 83–88.

Paper OaM-05-11

[15] A. LAMBERT AND S. PETROVIC, *Beyond hyperinvariance for compact operators*, J. Funct. Anal., **219** (2005), 93–108.
[16] A. BISWAS, S. PETROVIC, *On extended eigenvalues of operators*, Integral Equations and Operator Theory, **55** (2006), 233–248.
[17] S. PETROVIC, *On the extended eigenvalues of some Volterra operators*, Integral Equations and Operator Theory, **57** (2007), 593–598.
[18] S. SHKARIN, *Compact operators without extended eigenvalues*, J. Math. Anal. Appl., **332** (2007), 455–462.