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FACTORING IN THE METAPLECTIC GROUP AND OPTICS

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Abstract. The metaplectic group is generated by the Fourier transform and multiplications by functions of particular exponential type. Based on the use of the metaplectic representation and a factorization of symplectic matrices, in this paper a bound on the number of terms needed to factor an arbitrary metaplectic operator is derived. The approach is constructive and numerically stable, leading to a reliable factorization algorithm in practice. The problem is partially motivated by the task of constructing lens systems in diffractive optics.

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