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STRONGLY APPROXIMATIVE SIMILARITY OF A DENSE CLASS OF OPERATORS

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Abstract. Two operators A, B on a complex separable Hilbert space $\mathscr{H}$ are said to be strongly approximatively similar, denoted by $A \sim_{sas} B$, if (i) given $\varepsilon > 0$, there exist compact operators K_i with $\|K_i\| < \varepsilon (i = 1,2)$ such that $A + K_1$ and $B + K_2$ are similar; and (ii) $\sigma_0(A) = \sigma_0(B)$ and $\dim \mathscr{H}(\lambda; A) = \dim \mathscr{H}(\lambda; B)$ for each $\lambda \in \sigma_0(A)$. In this paper, we characterize strongly approximative similarity for a class of operators which is dense in $\mathscr{B}(\mathscr{H})$ in the operator norm. As a result, we infer that the relation $\sim_{sas}$ is an equivalence relation for this class of operators. A corresponding classification is accordingly obtained.

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