IRREDUCIBLE WAVELET REPRESENTATIONS AND ERGODIC AUTOMORPHISMS ON SOLENOIDS

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Abstract. We focus on the irreducibility of wavelet representations. We present some connections between the following notions: covariant wavelet representations, ergodic shifts on solenoids, fixed points of transfer (Ruelle) operators and solutions of refinement equations. We investigate the irreducibility of the wavelet representations, in particular the representation associated to the Cantor set, introduced in [13], and we present several equivalent formulations of the problem.

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