Operators and Matrices
Volume 5, Number 2 (2011), 227–237

A RESULT ON POSITIVE MATRICES AND APPLICATIONS TO HANKEL OPERATORS

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Abstract. Let S denote the shift operator on $l^2(\mathbb{N})$ and set $e_0 = (1,0,0,\dots)$. A special case of the main result says that if W is a self-adjoint operator on $l^2(\mathbb{N})$ such that $W(e_0) = 0$ and $S^*WS \geqslant W$, then $W \geqslant 0$. We apply this result to AAK-type theorems on generalized Hankel operators, providing new insights related to previous work by S. Treil and A. Volberg [10].

Mathematics subject classification (2010): Primary 47B35, Secondary 47B15, 47B37.
Keywords and phrases: Positive matrices, Hankel operators.

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Paper OaM-05-16