

ON THE BEST CONSTANTS IN MARKOV–TYPE INEQUALITIES INVOLVING GEGENBAUER NORMS WITH DIFFERENT WEIGHTS

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Abstract. The paper is concerned with best constants in Markov-type inequalities between the norm of a higher derivative of a polynomial and the norm of the polynomial itself. The norm of the polynomial is taken in L^2 with the Gegenbauer weight corresponding to a parameter α , while the derivative is measured in L^2 with the Gegenbauer weight for a parameter β . Under the assumption that $\beta - \alpha$ is an integer, we determine the first order asymptotics of the best constants as the degree of the polynomial goes to infinity.

Mathematics subject classification (2010): Primary 41A44; Secondary 15A18, 26D10, 45D05, 47B35.

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