

## UNITARY EQUIVALENCE TO A COMPLEX SYMMETRIC MATRIX: A MODULUS CRITERION

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*Abstract.* We develop a procedure for determining whether a square complex matrix is unitarily equivalent to a complex symmetric (i.e., self-transpose) matrix. We compare our approach to several existing methods [1, 19, 20] and present a number of examples.

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### REFERENCES

- [1] L. BALAYAN, S. R. GARCIA, *Unitary equivalence to a complex symmetric matrix: geometric criteria*, Operators and Matrices, **4**, 1 (2010), 53–76.
- [2] I. CHALENDAR, E. FRICAIN, D. TIMOTIN, *On an extremal problem of Garcia and Ross*, Operators and Matrices, **3**, 4 (2009), 541–546.
- [3] N. CHEVROT, E. FRICAIN, D. TIMOTIN, *The characteristic function of a complex symmetric contraction*, Proc. Amer. Math. Soc., **135**, 9 (2007), 2877–2886.
- [4] J. A. CIMA, S. R. GARCIA, T. W. ROSS, W. R. WOGEN, *Truncated Toeplitz operators: spatial isomorphism, unitary equivalence, and similarity*, Indiana Univ. Math. J., **59**, 2 (2010), 595–620.
- [5] J. A. CIMA, W. T. ROSS, W. R. WOGEN, *Truncated Toeplitz operators on finite dimensional spaces*, Operators and Matrices, **2**, 3 (2008), 357–369.
- [6] S. R. GARCIA, *Conjugation and Clark operators*, Contemp. Math., **393** (2006), 67–111.
- [7] S. R. GARCIA, M. PUTINAR, *Complex symmetric operators and applications*, Trans. Amer. Math. Soc., **358** (2006), 1285–1315.
- [8] S. R. GARCIA, M. PUTINAR, *Complex symmetric operators and applications II*, Trans. Amer. Math. Soc., **359**, 8 (2007), 3913–3931.
- [9] S. R. GARCIA, J. E. TENER, *On a problem of Halmos: unitary equivalence of a matrix to its transpose*, J. Operator Theory, to appear.
- [10] S. R. GARCIA, W. R. WOGEN, *Some new classes of complex symmetric operators*, Trans. Amer. Math. Soc., **362** (2010), 6065–6077.
- [11] S. R. GARCIA, W. R. WOGEN, *Complex symmetric partial isometries*, J. Funct. Anal., **257**, 4 (2009), 1251–1260.
- [12] T. M. GILBREATH, W. R. WOGEN, *Remarks on the structure of complex symmetric operators*, Integral Equations Operator Theory, **59**, 4 (2007), 585–590.
- [13] P. R. HALMOS, *A Hilbert Space Problem Book* (Second Edition), Springer-Verlag, New York, 1982.
- [14] P. R. HALMOS, *Linear Algebra Problem Book*, The Dolciani Mathematical Expositions, 16. Mathematical Association of America, Washington, DC, 1995.
- [15] R. A. HORN, C. R. JOHNSON, *Matrix Analysis*, Cambridge Univ. Press, Cambridge, 1985.
- [16] A. MCINTOSH, *The Toeplitz-Hausdorff theorem and ellipticity conditions*, Amer. Math. Monthly, **85**, 6 (1978), 475–477.
- [17] D. SARASON, *Algebraic properties of truncated Toeplitz operators*, Oper. Matrices, **1**, 4 (2007), 491–526.
- [18] T. Y. TAM, personal communication.

- [19] J. TENER, *Unitary equivalence to a complex symmetric matrix: an algorithm*, J. Math. Anal. Appl., **341**, 1 (2008), 640–648.
- [20] J. VERMEER, *Orthogonal similarity of a real matrix and its transpose*, Linear Algebra Appl., **428**, 1 (2008), 382–392.
- [21] X. WANG, Z. GAO, *A note on Aluthge transforms of complex symmetric operators and applications*, Integral Equations and Operator Theory, **65**, 4 (2009), 573–580.
- [22] S. M. ZAGORODNYUK, *On a  $J$ -polar decomposition of a bounded operator and matrix representations of  $J$ -symmetric,  $J$ -skew-symmetric operators*, Banach J. Math. Anal., **4**, 2 (2010), 11–36.