ON 2×2 OPERATOR MATRICES

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Abstract. In this paper, we show that some 2×2 operator matrices have scalar extensions. In particular, we focus on some 2-hyponormal operators and their generalizations. As a corollary, we get that such operator matrices have nontrivial invariant subspaces if their spectra have nonempty interiors in the complex plane.

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