Operators and Matrices
Volume 5, Number 3 (2011), 389–407

ESTIMATING EIGENVALUES OF MATRICES BY INDUCED NORMS

CARSTEN MICHELS

Abstract. A classical result of König in terms of matrices states that for $1 \leqslant p < q \leqslant \infty$ the eigenvalues $\lambda_1(A), \dots, \lambda_n(A)$ of an $n \times n$ square matrix A satisfy $\max_k k^{\frac{1}{p}-\frac{1}{q}} |\lambda_k(A)| \leqslant C_{q,p} \|A\|_{q,p}$ for some absolute constant $C_{q,p} > 0$ not depending on the matrix A, where $\|A\|_{q,p}$ denotes the norm of A viewed as an operator from ℓ_q^n into ℓ_p^n. We refine this result for $1 \leqslant p < q \leqslant 2$ by means of interpolation of Banach spaces.

Mathematics subject classification (2010): Primary: 15A60, secondary: 15A42, 65F35, 46B70, 47B06, 47B10.

Keywords and phrases: Eigenvalues, singular values, induced matrix norm, inequalities, interpolation of Banach spaces, absolutely summing norms.

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