UNIVERSAL SHIFTS AND COMPOSITION OPERATORS

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Abstract. It is shown that a large class of weighted shift operators $T$ have the property that for every $\lambda$ in the interior of the spectrum of $T$ the operator $U = T - \lambda \Id$ is universal in the sense of Caradus; i.e., every Hilbert space operator has a non-zero multiple similar to the restriction of $U$ to an invariant subspace. As an application, composition operators induced by power mappings on the $L^2$ and Sobolev spaces of the unit interval are shown to have the same property: thus a complete knowledge of their minimal invariant subspaces would imply a solution to the invariant subspace problem for Hilbert space. A new Müntz-like theorem is proved: this is used to show that generalized polynomials are cyclic vectors for these operators in the $L^2$ case but not in the Sobolev case.


Keywords and phrases: Invariant subspace, universal operator, weighted shift, composition operator, cyclic vector, Müntz theorem.

REFERENCES