

## UNIVERSAL SHIFTS AND COMPOSITION OPERATORS

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*Abstract.* It is shown that a large class of weighted shift operators  $T$  have the property that for every  $\lambda$  in the interior of the spectrum of  $T$  the operator  $U = T - \lambda \text{Id}$  is universal in the sense of Caradus; i.e., every Hilbert space operator has a non-zero multiple similar to the restriction of  $U$  to an invariant subspace. As an application, composition operators induced by power mappings on the  $L^2$  and Sobolev spaces of the unit interval are shown to have the same property: thus a complete knowledge of their minimal invariant subspaces would imply a solution to the invariant subspace problem for Hilbert space. A new Müntz-like theorem is proved: this is used to show that generalized polynomials are cyclic vectors for these operators in the  $L^2$  case but not in the Sobolev case.

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### REFERENCES

- [1] P. BORWEIN AND T. ERDÉLYI, *The full Müntz theorem in  $C[0, 1]$  and  $L_1[0, 1]$* , J. London Math. Soc. (2), **54**, 1 (1996), 102–110.
- [2] S. R. CARADUS, *Universal operators and invariant subspaces*, Proc. Amer. Math. Soc., **23**, 1 (1969), 526–527.
- [3] I. CHALENDAR AND J. ESTERLE, *Le problème du sous-espace invariant*, Development of mathematics 1950–2000, pages 235–267, Birkhäuser, Basel, 2000.
- [4] E. NORDGREN, P. ROSENTHAL, AND F. S. WINTROBE, *Invertible composition operators on  $H^p$* , J. Funct. Anal., **73**, 2 (1987), 324–344.
- [5] W. C. RIDGE, *Approximate point spectrum of a weighted shift*, Trans. Amer. Math. Soc., **147** (1970), 349–356.
- [6] H. RADJAVI AND P. ROSENTHAL, *Invariant subspaces*, Dover Publications Inc., Mineola, NY, second edition, 2003.
- [7] A. SPALSBURY, *Perturbations in Müntz’s theorem*, J. Approx. Theory, **150**, 1 (2008), 48–68.