

## HYPERINVARIANT SUBSPACES FOR OPERATORS HAVING A NORMAL PART

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*Abstract.* Let  $T$  be a nonscalar operator of the form  $\begin{pmatrix} A & * \\ 0 & B \end{pmatrix}$ . It is well known ([5], [6]) that if both  $A$  and  $B$  are normal operators, then  $T$  has a nontrivial hyperinvariant subspace. In this paper, it is shown that if  $A$  is a nonscalar normal operator, then either  $\begin{pmatrix} A & C \\ 0 & B \end{pmatrix}$  or  $\begin{pmatrix} A & 0 \\ D & B \end{pmatrix}$  has a nontrivial hyperinvariant subspace.

*Mathematics subject classification (2010):* 47A15.

*Keywords and phrases:* Hyperinvariant subspaces, normal operators, extremal vectors.

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