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SELF–INVERSIVE MATRIX POLYNOMIALS WITH SEMISIMPLE SPECTRUM ON THE UNIT CIRCLE

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Abstract. The spectrum of a class of self-inversive matrix polynomials is studied. It is shown that the characteristic values are semisimple and lie on the unit circle if the inner radius of an associated matrix polynomial is greater than 1.

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