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$\mathbb{C}$–ORBIT REFLEXIVE OPERATORS

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Abstract. We introduce the notion of $\mathbb{C}$-orbit reflexivity and study its properties. An operator on a finite-dimensional space is $\mathbb{C}$-orbit reflexive if and only if the two largest blocks in its Jordan form corresponding to nonzero eigenvalues with the largest modulus differ in size by at most one. Most of the proofs of our results in infinite dimensions are obtained from purely algebraic results we obtain from linear-algebraic analogs of $\mathbb{C}$-orbit reflexivity.

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