

THE ORTHOGONALITY STRUCTURE DETERMINES A C^* -ALGEBRA WITH CONTINUOUS TRACE

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Abstract. There are four versions of disjointness structures of a C^* -algebra: zero product, range orthogonality, domain orthogonality and doubly orthogonality. Recently, Leung and Wong show that the linear and zero product structures are sufficient to determine the CCR C^* -algebras with Hausdorff spectrums. In this paper, we investigate the orthogonality structures of the C^* -algebras. More precisely, let θ be a bijective linear map between two C^* -algebras with continuous traces. We prove that θ is automatically continuous whenever it preserves range (respectively, domain) orthogonal elements in both senses.

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