Operators and Matrices
Volume 5, Number 3 (2011), 541–551

JORDAN $*$–HOMOMORPHISMS ON C^*–ALGEBRAS

M. ESHAGHI GORDJI, N. GHOBADIPOUR AND CHOONKIL PARK

Abstract. In this paper, we investigate Jordan $*$-homomorphisms on C^*-algebras associated with the following functional inequality $\left\|f\left(\frac{b-a}{3}\right)+f\left(\frac{a-3c}{3}\right)+f\left(\frac{3a+3c-b}{3}\right)\right\| \leqslant \|f(a)\|$. We moreover prove the superstability and the generalized Hyers-Ulam stability of Jordan $*$-homomorphisms on C^*-algebras associated with the following functional equation

$$f\left(\frac{b-a}{3}\right)+f\left(\frac{a-3c}{3}\right)+f\left(\frac{3a+3c-b}{3}\right)=f(a).$$

Mathematics subject classification (2010): 17C65, 39B82, 46L05, 47Jxx, 47B48, 39B72.

Keywords and phrases: Jordan $*$-homomorphism, C^*-algebra, generalized Hyers-Ulam stability, functional equation and inequality.

REFERENCES

[1] B. BAAK, D. BOO AND TH. M. RASSIAS, *Generalized additive mapping in Banach modules and isomorphisms between C*-algebras*, J. Math. Anal. Appl., **314** (2006), 150–161.

[2] P. W. CHOLEWA, *Remarks on the stability of functional equations*, Aequationes Math., **27** (1984), 76–86.

[3] JAE YOUNG CHUNG, *Distributional methods for a class of functional equations and their stabilities*, Acta Mathematica Sinica, English Series, **23**, 11 (2007), 2017–2026.

[4] Z. GAJDA, *On stability of additive mappings*, Internat. J. Math. Math. Sci., **14** (1991), 431–434.

[5] P. GĂVRUTA, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl., **184** (1994), 431–436.

[6] D. H. HYERS, *On the stability of the linear functional equation*, Proc. Nat. Acad. Sci. USA, **27** (1941), 222–224.

[7] D. H. HYERS, G. ISAC AND TH. M. RASSIAS, *Stability of Functional Equations in Several Variables*, Birkhauser, Basel, 1998.

[8] D. H. HYERS AND TH. M. RASSIAS, *Approximate homomorphisms*, Aequationes Math., **44** (1992), 125–153.

[9] G. ISAC AND TH. M. RASSIAS, *On the Hyers-Ulam stability of ψ-additive mappings*, J. Approx. Theory, **72** (1993), 131–137.

[10] G. ISAC AND TH. M. RASSIAS, *Stability of ψ-additive mappings: Applications to nonlinear analysis*, Internat. J. Math. Math.Sci., **19** (1996), 219–228.

[11] KIL-WOUNG JUN AND HARK-MAHN KIM, *Stability problem for Jensen–type functional equations of cubic mappings*, Acta Mathematica Sinica, English Series, **22**, 6 (2006), 1781–1788.

[12] K. JUN AND Y. LEE, *A generalization of the Hyers-Ulam-Rassias stability of Jensen's equation*, J. Math. Anal. Appl., **238** (1999), 305–315.

[13] S. JUNG, *Hyers-Ulam-Rassias stability of Jensen's equation and its application*, Proc. Amer. Math. Soc., **126** (1998), 3137–3143.

[14] HARK-MAHN KIM, *Stability for generalized Jensen functional equations and isomorphisms between C^*-algebras*, Bulletin of the Belgian Mathematical Society - Simon Stevin, **14**, 1 (2007), 1–14.

[15] BYUNG DO KIM, *On the derivations of semiprime rings and noncommutative Banach algebras*, Acta Mathematica Sinica, English Series, **16**, 1 (2000), 21–28.

[16] BYUNG DO KIM, *On Hyers–Ulam–Rassias stability of functional equations*, Acta Mathematica Sinica, English Series, **24**, 3 (2008), 353–372.

Paper OaM-05-41

[17] MOHAMMAD SAL MOSLEHIAN, *Almost Derivations on C^*-Ternary Rings*, Bulletin of the Belgian Mathematical Society - Simon Stevin, **14**, 1 (2007), 135–142.
[18] C. PARK, *Homomorphisms between Poisson JC*-algebras*, Bull. Braz. Math. Soc., **36** (2005), 79–97.
[19] C. PARK, J. AN AND J. CUI, *Jordan $*$-derivations on C^*-algebras and JC^*-algebras*, Abstact and Applied Analasis, in press.
[20] CHUN-GIL PARK, *Hyers-Ulam-Rassias stability of a generalized Euler-Lagrange type additive mapping and isomorphisms between C^*-algebras*, Bulletin of the Belgian Mathematical Society - Simon Stevin, **13**, 4 (2006), 619–631.
[21] CHOONKIL PARK AND JIAN LIAN CUI, *Approximately linear mappings in Banach modules over a C *-algebra*, Acta Mathematica Sinica, English Series, **23**, 11 (2007), 1919–1936.
[22] TH. M. RASSIAS, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., **72** (1978), 297–300.
[23] TH. M. RASSIAS, *Approximate homomorphisms*, Aequationes Math., **44** (1992), 125–153.
[24] TH. M. RASSIAS, *On the stability of functional equations and a problem of Ulam*, Acta Math. Appl., **62** (2000), 23–130. MR1778016 (2001j:39042)
[25] TH. M. RASSIAS, *On the stability of functional equations in Banach spaces*, J. Math. Anal. Appl., **251** (2000), 264–284.
[26] P. K. SAHOO, *A generalized cubic functional equation*, Acta Mathematica Sinica, English Series, **21**, 5 (2005), 1159–1166.
[27] S. M. ULAM, *Problems in Modern Mathematics*, Chapter VI, Science ed. Wiley, New York, 1940.
[28] DENG HUA ZHANG AND HUAI XIN CAO, *Stability of functional equations in several variables*, Acta Mathematica Sinica, English Series, **23**, 2 (2007), 321–326.