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INDEFINITE BOUNDARY VALUE PROBLEMS ON GRAPHS

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Abstract. We consider the spectral structure of indefinite second order boundary-value problems on graphs. A variational formulation for such boundary-value problems on graphs is given and we obtain both full and half-range completeness results. This leads to a max-min principle and as a consequence we can formulate an analogue of Dirichlet-Neumann bracketing and this in turn gives rise to asymptotic approximations for the eigenvalues.

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