Operators and Matrices
Volume 5, Number 4 (2011), 585–617

ON A COMMUTATIVE WJ^*–ALGEBRA OF D_1^+–CLASS AND ITS BICOMMUTANT

VLADIMIR STRAUSS

Abstract. We study different properties of a commutative WJ^*-algebra in a Krein space that has a maximal non-negative subspace represented as a direct sum of its one-dimensional isotropic subspace and a uniformly positive one. In particulary we give a criteria for the equality between of a WJ^*-algebra of this class and its bicommutant.

Mathematics subject classification (2010): Primary 46C20, 46K99, 47B50; secondary 47B40, 47A60.
Keywords and phrases: Indefinite metric, operator algebras, model representation, functional calculus, bicommutant.

REFERENCES

[1] N. I. AKHIEZER, I. M. GLAZMAN, *Theory of linear operators in Hilbert space*, Pitman: London, 1981.
[2] T. YA. AZIZOV, I. S. IOKHVIDOV, *Foundation of the Theory of Linear Operators in Spaces with Indefinite Metric*, Nauka, Moskow, 1986 (in Russian); Linear Operators in Spaces with Indefinite Metric, Wiley, New York, 1989.
[3] T. YA. AZIZOV, V. A. STRAUSS, *Spectral decompositions for special classes of self-adjoint and normal operators on Krein spaces*, Spectral Theory and its Applications, Proceedings dedicated to the 70-th birthday of Prof. I.Colojoară, Theta 2003, 45–67.
[4] T. YA. AZIZOV, V. A. STRAUSS, *On a spectral decomposition of a commutative operator family in spaces with indefinite metric.* MFAT **11**, 1 (2005), 1–20.
[5] O. YA. BENDERSKY, S. N. LITVINOV AND V. I. CHILIN, *A description of commutative symmetric operator algebras in a Pontryagin space* Π_1, Preprint, Tashkent 1989 (Russian).
[6] O. YA. BENDERSKY, S. N. LITVINOV AND V. I. CHILIN, *A description of commutative symmetric operator algebras in a Pontryagin space* Π_1, Journal of Operator Theory **37**, 2 (1997), 201–222.
[7] J. BERGH, J. LÖFSTRÖM, *Interpolation spaces. An Introduction*, Springer Verlag, NY, 1976.
[8] M. S. BIRMAN, M. Z. SOLOMYAK, *Spectral theory of self-ajoint operators in a Hilbert space*, Leningrad University, 1980 (Russian).
[9] O. BRATTELI, D. W. ROBINSON, *Operator Algebras and Quantum Statistical Mechanics*, Vol. **1**, Springer-Verlag, NY, 1979.
[10] N. DUNFORD, J. T. SCHWARTZ, *Linear Operators. Part III. Spectral operators*, John Wiley & Sons, 1971.
[11] A. GHEONDEA, *Pseudo-regular spectral functions in Krein spaces*, J. of Oper.Th. **12** (1984), 349–358.
[12] R. S. ISMAGILOV, M. A. NAIMARK, *Representations of groups and algebras in spaces with indefinite metric,* "Matematicheskii analiz" (Itogi nauki i tehniki 1968, VINITI) (1969), Moscow, 73–105 (Russian).
[13] P. JONAS, H. LANGER, *A model for π-selfadjont operators in π_1-spaces and a special linear pencil*, Integr. Equat. and Oper.Theory **8**, 1 (1985), 13–35.
[14] E. KISSIN, V. SHULMAN, *Representations of Krein spaces and derivations of C^*-algebras*, Pitman Monographs and Surveys in Pure and Applied Mathematics **89**, Addison-Vesely Longman, 1997.
[15] S. G. KREIN, YU. I. PETUNIN, YE. M. SEMYONOV, *Interpolation of linear operators*, Nauka, Moscow, 1978 (Russian).

Paper OaM-05-44

[16] H. LANGER, B. TEXTORIUS, *L-resolvent matrices of symmetric linear relations with equal defect numbers; applications to canonical differential relations*, Integr.Equat. and Oper.Th. **5**, 2 (1982), 208–243.

[17] S. N. LITVINOV, *Description of commutative symmetric algebras in the Pontryagin space Π_1*, DAN UzSSR 1 (1987), 9–12 (Russian).

[18] A. I. LOGINOV, *Complete commutative symmetric algebras in Pontryagin spaces Π_1*, Mat. Sbornik **84**, 4 (1971), 575–582 (Russian).

[19] M. A. NAIMARK, *Commutative algebras of operators in a space Π_1*, Rev. roum. math. pures et appl. **9**, 6 (1964), 499-529 (Russian).

[20] M. A. NAIMARK, *Normed Algebras*, Wolters-Nordhoff Publishing, Groningen, The Nedherlands, 1972.

[21] M. REED, B. SIMON, *Methods of Modern Mathematical Physics. I: Functional Analysis*, 2nd Edition, Acad. Press, Inc., 1980.

[22] V. A. SHTRAUS (=STRAUSS), *On the structure of operators that doubly commute with operators of the class $K(H)$* (Russian), Ukrain. Mat. Zh. **38**, 6 (1986), 805–817.

[23] V. A. STRAUSS, *Functional representation of operators that doubly commute with a selfadjoint operator in a Pontryagin space* (Russian), Sibirsk. Mat. Zh. **29**, 6 (1988), 176–184; translation in Siberian Math. J. **29**, 6 (1988), 1012–1018 (1989).

[24] V. A. STRAUSS, *The structure of a family of commuting J-self-adjoint operators*, Ukrain. Mat.Zh. **41**, 10 (1989), 1431–1433, 1441 (Russian).

[25] V. STRAUSS, *On the bicommutant for one type of J-symmetric nilpotent algebras in Krein spaces*, Linear Algebra and Applications **372** (2003), 167–180.

[26] V. STRAUSS, *A functional description for the commutative WJ^*-algebras of the D_κ^+-class*, Proceedings of Colloquium on Operator Theory and its Applications dedicated to Prof. Heinz Langer (Vienna, 2004), in Operator Theory: Advances and Applications **163** (2005), Birkhäuser Verlag, 299–335.

[27] V. STRAUSS, *On models of function type for a special class of normal operators in Krein spaces and their polar representation*, MFAT **13**, 1 (2007), 67–82.

[28] V. STRAUSS, *Models of function type for commutative symmetric operator families in Krein spaces*, Abst. and Appl. Analysis **2008**, Article ID 439781, 40 pages, 2008.

[29] V. STRAUSS, *On spectral functions for commutative J-self-adjoint operator families of D_κ^+-class*, Contemporary Mathematics **455**, Amer. Math. Soc., Providence, RI, 2008, 349–367.

[30] V. S. SHULMAN, *Symmetric Banach algebras of operators in a space of type Π_1*, Mat. Sb. (N.S.) **89(131)**, 2 (1972), 264–279 (Russian).