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# NON–DEFINITE STURM–LIOUVILLE PROBLEMS FOR THE $p$–LAPLACIAN

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*Abstract.* For a weighted Sturm-Liouville-type problem of the form

$$-\Delta_p y = (p-1)(\lambda r - q)\operatorname{sgn} y|y|^{p-1}, \quad \text{on } (0,1)$$

with Sturmian-type boundary conditions ($\Delta_p$ being the $p$-Laplacian), we examine the structure, asymptotics and parametric dependence of the eigenvalues, together with properties of the eigenfunctions such as oscillation and interlacing of zeros. We discuss definitions and consequences of left and right (semi-) definiteness, and also the fully indefinite case.

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## REFERENCES

[1] A. ANANE, O. CHAKRONE, M. MOUSSA, *Spectrum of one dimensional p-Laplacian Operator with indefinite weight*, EJQTDE **17** (2002), 1–11.
[2] W. ALLEGRETTO, *Principal eigenvalues for indefinite-weight elliptic problems in* $\mathbb{R}^N$, Proc. Amer. Math. Soc. **116** (1992), 701–706.
[3] F.V. ATKINSON, A. MINGARELLI, *Asymptotics of the number of zeros and of the eigenvalues of general weighted Sturm-Liouville problems*, J. Reine Angew. Math. **375/376** (1987), 380–393.
[4] P. BINDING, P. J. BROWNE, *Left definite Sturm-Liouville problems with eigenparameter dependent boundary conditions*, Differential Integral Equations **12** (1999), 167–182.
[5] P. BINDING, P. DRÁBEK, *Sturm-Liouville theory for the p-Laplacian*, Studia Scientiarum Math. Hungarica **40** (2003), 375–396.
[6] P. A. BINDING, Y. HUANG, *Existence and non-existence of positive eigenfunctions for the p-Laplacian*, Proc. Amer. Math. Soc. **123** (1995), 1833–1838.
[7] P. BINDING, H. VOLKMER, *Prüfer angle asymptotics for Atkinson's semi-definite Sturm-Liouville eigenvalue problem*, Math. Nachr. **278** (2005), 1458–1475.
[8] P. BINDING, H. VOLKMER, *Bounds for the eigenvalues of the p-Laplacian with weight function of bounded variation*, Op. Theory Adv. Appl. **203** (2010), 99–114.
[9] J. BONDER, J. PINASCO, *Eigenvalues of the p-Laplacian in fractal strings with indefinite weights*, J. Math. Anal. Appl. **308** (2005), 764–774.
[10] M. CUESTA, H. RAMOS QUOIRIN, *A weighted eigenvalue problem for the p-Laplacian plus a potential*, Nonlinear Differential Equations Appl. **16** (2009), 469–491.
[11] L. DEL PEZZO, J. BONDER, *An optimization problem for the first eigenvalue of the p-Laplacian plus a potential*, Commun. Partial Differ. Equ. **5** (2006), 675–690.
[12] A. DOŠLY, P. ŘEHÁK, Half-linear Differential Equations, North Holland Math. Studies **202**, Elsevier, 2005.
[13] A. ELBERT, *A half-linear second order differential equation*, Colloquia Mathematica Societatis Janos Bolyai **30**, Qualitative Theory of Differential Equations, 1979, pp. 124–143.
[14] W. EBERHARD, A. ELBERT *On the eigenvalues of a half-linear boundary value problem*, Math. Nachr. **213** (2000), 57–76.
[15] W. N. EVERITT, M. KWONG, A. ZETTL *Oscillation of eigenfunctions of weighted regular Sturm-Liouville problems*, J. London Math. Soc. **27** (1983), 106–120.


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[16] E. INCE, *Ordinary Differential Equations*, Dover Publishers, New York, 1993.
[17] T. KUSANO, M. NAITO, *Sturm-Liouville eigenvalue problems from half-linear ordinary differential equations*, Rocky Mountain J. Math. **31** (2001), 1039–1054.
[18] G. MENG, P. YAN, M. ZHANG *Spectrum of one-dimensional $p$-Laplacian with an indefinite integrable weight*, Mediterr. J. Math. **7** (2010), 225–248.
[19] M. A. NAIMARK, *Linear Differential Operators I,* Ungar, New York, 1967.
[20] R. RICHARDSON, *Theorems of oscillation for two linear differential equations of the second order with two parameters*, Trans. Amer. Math. Soc. **13** (1912), 22–34.
[21] W. REID *Ordinary Differential Equations*, Wiley, New York, 1971.
[22] W. REICHEL, W. WALTER, *Sturm-Liouville type problems for the $p$-Laplacian under asymptotic non-resonance conditions*, J. Differential Equations **156** (1999), 50–70.
[23] W. WALTER, *Ordinary Differential Equations*, Springer, New York, 1998.
[24] A. ZETTL, Sturm-Liouville Theory, Amer. Math. Soc., Mathematical Surveys and Monographs **121**, 2005.