

ON SPLITTINGS OF MATRICES AND NONNEGATIVE GENERALIZED INVERSES

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Abstract. The authors introduce a new type of matrix splitting generalizing the notion of B -splitting and study its relationships with nonnegativity of the Moore-Penrose inverse and the group inverse.

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