STRONG COMMUTATIVITY PRESERVING MAPS ON TRIANGULAR RINGS

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Abstract. Let \( \mathcal{U} = \text{Tri}(\mathcal{A}, \mathcal{M}, \mathcal{B}) \) be a triangular ring. It is shown, under some mild assumption, that every surjective strong commutativity preserving map \( \Phi: \mathcal{U} \to \mathcal{U} \) (i.e. \( \Phi(T), \Phi(S) = [T, S] \) for all \( T, S \in \mathcal{U} \)) is of the form \( \Phi(T) = ZT + f(T) \), where \( Z \in Z(\mathcal{U}) \), the center of \( \mathcal{U} \), \( Z^2 = I \) and \( f \) is a map from \( \mathcal{U} \) into \( Z(\mathcal{U}) \). As an application, a characterization of general surjective maps that preserve the strong commutativity on the nest algebras of Banach space operators is given.


Keywords and phrases: triangular rings, nest algebras, strong commutativity, general preservers.

REFERENCES