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ON MINIMAL POTENTIALLY POWER–POSITIVE SIGN PATTERNS

BER-LIN YU AND TING-ZHU HUANG

Abstract. An n-by-n sign pattern $\mathscr{A}$ is said to be potentially power-positive if there exists some $A \in Q(\mathscr{A})$ such that A is power-positive, i.e., $A^k > 0$ for some positive integer k. Catral, Hogben, Olesky and van den Driessche [Sign patterns that require or allow power-positivity, *Electron. J. Linear Algebra*, 19 (2010), 121-128] investigated the sign patterns that require or allow power-positivity. It has been shown that an n-by-n sign pattern $\mathscr{A}$ is potentially power-positive if and only if either $\mathscr{A}$ or $-\mathscr{A}$ is potentially eventually positive. But as the identification of sufficient and necessary conditions for potentially eventually positive sign patterns remains open, the characterization of potentially power-positive sign patterns is still open. In this paper, we introduce the minimal potentially power-positive sign patterns to classify the potentially power-positive sign patterns. Some properties of minimal potentially power-positive sign patterns are presented. It is shown that for an n-by-n sign pattern $\mathscr{A}$ with at most $n+1$ negative entries, $\mathscr{A}$ is minimal potentially power-positive if and only if either $\mathscr{A}$ or $-\mathscr{A}$ is minimal potentially eventually positive. Finally, we classify the minimal potentially power-positive sign patterns of order $n \leqslant 3$.

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