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WHEN DOES THE MOORE–PENROSE INVERSE FLIP?

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Abstract. In this paper, we give necessary and sufficient conditions for the matrix $\begin{bmatrix} a & 0 \\ b & d \end{bmatrix}$, over a *-regular ring, to have a Moore-Penrose inverse of four different types, corresponding to the four cases where the zero element can stand. In particular, we study the case where the Moore-Penrose inverse of the matrix flips.

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