

FREDHOLM STABILITY RESULTS FOR LINEAR COMBINATIONS OF m -POTENT OPERATORS

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Abstract. We investigate the stability of the nullity, defect and index of linear combinations $uA + vB$ of generalized quadratic operators, extending in several directions the recent results of Gau, Wang and Wong (*Oper. Matrices* **2** (2008), 193–199), and of commuting scalar m -potent operators. For the latter case we prove that the nullspaces of $uA + vB$ themselves are also stable.

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