

## M-IDEALS AND THE BISHOP-PHELPS THEOREM

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*Abstract.* We give new proofs for the known important approximative properties of M-ideals, using only the definition of an M-ideal and the Bishop-Phelps theorem. Unlike the known proofs, these proofs do not use the 3-ball intersection property of M-ideals.

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### REFERENCES

- [1] E. BEHREND, *M-Structure and the Banach-Stone theorem*, Lecture Notes in Math. 736, Springer, Berlin-Heidelberg-New York, 1979.
- [2] E. BEHREND AND P. HARMAND, *Banach spaces which are proper M-ideals*, *Studia Math.* **81** (1985), 159–169.
- [3] E. BISHOP AND R. R. PHELPS, *The support functionals of a convex set*, *Convexity* (Victor L. Klee, ed.), Proc. of symposia in Pure Math., Vol. 7, Amer. Math. Soc (1963), 27–35.
- [4] H. FAKHOURY, *Existence d'une projection continue de meilleur approximation dans certains espaces de banach*, *J. Math. pures et appl.* **53** (1974), 1–16.
- [5] G. GODEFROY AND V. INDUMATHI, *Norm-to-weak upper semi-continuity of the duality and Pre-duality mappings*, *Set-Valued analysis* **10** (2002), 317–330.
- [6] P. HARMAND, D. WERNER AND W. WERNER, *M-ideals in Banach spaces and Banach algebras*, Lecture Notes in Math. 1574, Springer, Berlin-Heidelberg-New York, 1993.
- [7] R. B. HOLMES, B. SCRANTON, AND J. WARD, *Approximation from the space of compact operators and other M-ideals*, *Duke Math.J.* **42** (1975), 259–269.
- [8] V. INDUMATHI AND S. LALITHAMBIGAI, *A new proof of proximality for M-ideals*, *Proc. Amer. Math. Soc.* **135**, 4 (2007), 1159–1162.
- [9] ASVALD LIMA, *On M-ideals and best approximation*, *Indiana Univ. Math. J.* **31** (1982), 27–36.
- [10] ROBERT E. MEGGINSON, *An introduction to Banach space theory*, Springer, 1988.
- [11] DAVID YOST, *Best approximation and intersection of balls in Banach spaces*, *Bull. Austral. Math. Soc.* **20** (1979), 285–300.