Operators and Matrices
Volume 6, Number 2 (2012), 241–262
doi:10.7153/oam-06-17

SPACES OF p–INTEGRABLE FUNCTIONS WITH RESPECT TO A VECTOR MEASURE DEFINED ON A δ–RING

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Abstract. In this paper we study the lattice properties of the Banach lattices $L^p(\nu)$ and $L^p_w(\nu)$ of p-integrable real-valued functions and weakly p-integrable real-valued functions with respect to a vector measure ν defined on a δ-ring. The relation between these two spaces, the study of the continuity and some kind of compactness properties of certain multiplication operators between different spaces L^p and/or L^q_w and the representation theorems of general Banach lattices via these spaces play a fundamental role.

Mathematics subject classification (2010): Primary 46G10, Secondary 46E30, 46B42.
Keywords and phrases: Banach lattice, δ-ring, Vector measure, Integration.

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Paper OaM-06-17

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