GEOMETRIC THEORY OF WEAK KOROVKIN SETS

M. N. N. NAMBOODIRI

Abstract. A set S of generators of an abstract W^*-algebra $\mathscr{A}$ is called weakly hyperrigid if for every faithful representation of $\mathscr{A}$ on a Hilbert space $\mathscr{A} \subseteq B(H)$ and every net $\{\Phi_\alpha\}_{\alpha \in I}$ of completely positive maps, of norm $\leqslant 1$, on $B(H)$,

$$\lim_\alpha \Phi_\alpha(s) = s \text{ weakly, for all } s \in S \Rightarrow \lim_\alpha \Phi_\alpha(a) = a \text{ weakly },$$

for all $a \in \mathscr{A}$. This is analogous to W. B. Arveson's [6] hyperrigid sets in C^* algebras. A characterisation of weakly hyperrigid sets and a noncommutative analogue of Y. Saskin's theorem [17] on geometric characterisation of Korovkin sets in $C(X)$ is proved.

Mathematics subject classification (2010): Primary 46L07; Secondary 46L52.
Keywords and phrases: Korovkin, W^*-algebras, completely positive maps.

REFERENCES

[1] F. ALTOMARE AND M. CAMPITI, *Korovkin type approximation theory and its applications*, de Gruyter Studies in Mathmatics, Berlin, New York, 1994.
[2] F. ALTOMARE, *Korovkin-type theorems and approximation by positive linear operators*, Surveys in Approximation Theory **5** (2010) 92–164.
[3] W. B. ARVESON, *Subalgebras of C^*-algebras*, Acta. Math. **123** (1969), 141–224.
[4] W. B. ARVESON, *Subalgebras of C^*-algebras II*, Acta. Math. **128** (1972), 271–308.
[5] W. B. ARVESON, *The noncommutative Choquet boundary I*, arXIV:MATH0701329 V4 1 [math OA], 19 Feb 2007.
[6] W. B. ARVESON, *The noncommutative Choquet boundary II: Hyperrigidity*, arXIV:08102751 V4 [math OA], 28 May 2009.
[7] H. BERENS AND G. G. LORENTZ, *Geometric theory of Korovkin sets*, J. Approx. Theory, Acad. Press 1975.
[8] P. P. KOROVKIN, *Linear operators and approximation theory*, Hindustan Publ. Corp. Delhi, India, 1960.
[9] B. V. LIMAYE AND M. N. N. NAMBOODIRI, *Korovkin-type approximation on C^* algebras*, J. Approx. Theory **34**, 3 (1982), 237–246.
[10] B. V. LIMAYE AND M. N. N. NAMBOODIRI, *Weak Korovkin approximation by completely positive maps on $B[H]$*, J. Approx. Theory, Acad. Press, 1984.
[11] B. V. LIMAYE AND M. N. N. NAMBOODIRI, *A generalized noncommutative Korovkin theorem and closedness of certain sets of convergence*, Ill. J. Math **28** (1984), 267–280.
[12] M. N. N. NAMBOODIRI, *Geometric formulation of noncommutative (weak) Korovkin sets*, Short talk given at ICM Satellite Conference on Functional Analysis and Operator Theory held at ISI Bangalore, August 07–10, 2010.
[13] OLA BRATTELE AND D. W. ROBINSON, *Operator algebras and quantum statistical mechanics-I, C^* and W^* algebras, Symmetry groups, Decomposition of states*, Springer-Verlag, New York-Heidelberg-Berlin, 1979.
[14] W. M. PRIESTLEY, *A noncommutative Korovkin theorem*, J. Approx. Theory **16**, 3 (1976), 251–260.
[15] A. G. ROBERTSON, *A Korovkin theorem for Schwartz maps on C^* algebras*, Math. Z. **156**, 2 (1977), 205–207.

[16] A. G. ROBERTSON, *Assymmetric invariants sets for completely positive maps on C^*-star algebras*, Bull. Austral. Math. Soc. **33**, 3 (1986), 471–473.

[17] Y. A. SASKIN, *Korovkin systems in spaces of continuous functions*, A. M. S. Translations [2] **54** (1966), 125–144.

[18] S. TAKAHASI, *Korovkin's theorem for C^*-algebras*, J. Approx. Theory **27** (1979).

[19] M. UCHIYAMA, *Korovkin type theorems for Schwartz maps and operator monotone functions in C^*-algebras*, Math. Z. **230** (1999).

[20] D. E. WULBERT, *Convergence of operators and Korovkin's theorem*, J. Approximation Theory **1** (1968), 381–390.