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CENTRAL AND ALMOST CONSTRAINED SUBSPACES OF BANACH SPACES

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Abstract. In this paper we continue the study of central subspaces initiated in [2] and its infinite version called almost constrained subspaces. We are interested in studying situations where these intersection properties of balls lead to the existence of a linear projection of norm one. We show that every finite dimensional subspace is a central subspace only in Hilbert spaces. By considering direct sums of Banach space we give examples where central subspaces are almost constrained or one-complemented. We show that a M-ideal can fail to be a central subspace, answering a question raised in [2].

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