BANACH ALGEBRAS OF OPERATOR SEQUENCES

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Abstract. During the last decades it turned out to be fruitful to apply certain Banach algebra techniques in the theory of approximation of operators by matrix sequences. Here we discuss the case of operator sequences (acting on infinite dimensional Banach spaces and which do not necessarily converge strongly) and we derive analogous results concerning the stability and Fredholm properties of such sequences. For this, the notions of $\mathscr{P}$-Fredholmness and $\mathscr{P}$-strong convergence play an important role and are extensively studied. As an application we consider the finite sections of band-dominated operators on l^p-spaces, including the cases $p \in \{1, \infty\}$.

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