

PRESERVERS OF MATRIX PAIRS WITH A FIXED INNER PRODUCT VALUE

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Abstract. Let \mathcal{V} be the set of $n \times n$ hermitian matrices, the set of $n \times n$ symmetric matrices, the set of all effects, or the set of all projections of rank one. Let c be a real number. We characterize bijective maps $\phi : \mathcal{V} \rightarrow \mathcal{V}$ satisfying $\operatorname{tr}(AB) = c \iff \operatorname{tr}(\phi(A)\phi(B)) = c$ with some additional restrictions on c , depending on the underlying set of matrices.

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