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ON AN INVERSE FORMULA OF A TRIDIAGONAL MATRIX

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Abstract. This paper provides an inverse formula freed of determinant expressions for a general tridiagonal matrix. This is viewed as an alternative version of the Usmani formula, which easily tends to blow up computationally. We discuss a number of different viewpoints regarding the proposed and Usmani's formulas, such as the proof method and the meaning of included terms, although our formula itself may be obtained by a simple transformation of Usmani's. A study of the limit elements based on the inverse formula and a numerical experiment for comparison with the other inverse methods are provided. In addition, we briefly discuss the inverse formula in the case of zero minors, which is illustrated by a numerical example.

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