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ALL–DERIVABLE POINTS OF NEST ALGEBRAS ON BANACH SPACES

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Abstract. Let $\mathscr{N}$ be a nest on a real or complex Banach space X and let $\mathrm{Alg}\mathscr{N}$ be the associated nest algebra. $\Omega \in \mathrm{Alg}\mathscr{N}$ is called an additively all-derivable point if for any additive map $\delta : \mathrm{Alg}\mathscr{N} \to \mathrm{Alg}\mathscr{N}$, $\delta(AB) = \delta(A)B + A\delta(B)$ holds for any $A, B \in \mathrm{Alg}\mathscr{N}$ with $AB = \Omega$ implies that δ is an additive derivation. Assume that P is an idempotent operator with range $\mathrm{ran}(P) = N_0$ for some nontrivial $N_0 \in \mathscr{N}$. Let $\Omega \in \mathrm{Alg}\mathscr{N}$ be any operator satisfying that $P\Omega P = \Omega$ (or $(I-P)\Omega(I-P) = \Omega$). We show that, if $\Omega|_{\mathrm{ran}(P)}$ (or $\Omega|_{\mathrm{ran}(I-P)}$) is injective or has dense range, then Ω is an additively all-derivable point. Moreover, if X is infinite dimensional, then every additive map derivable at such an Ω is an inner derivation.

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