IDEALS OF COMPACT OPERATORS WITH
NAKANO TYPE NORMS IN A HILBERT SPACE

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Abstract. Let H be a separable Hilbert space with a norm $\|.\|_H$. For a compact linear operator A acting in H, let $\lambda_k(A)$ be the eigenvalues, $s_k(A)$ ($k = 1, 2, ...$) singular values and $\|A\|_H = \sup_{x \in H} \|Ax\|_H / \|x\|_H$. Let $\pi = \{p_k\}_{k=1}^{\infty}$ be a nondecreasing sequence of numbers $p_k \geqslant 1$. Put

$$\gamma_\pi(A) := \sum_{j=1}^{\infty} \frac{s_j^{p_j}(A)}{p_j}.$$

We investigate the ideal X_π of operators satisfying $\gamma_\pi(tA) < \infty$ for all $t > 0$. In particular, it is proved that for any $A \in X_\pi$ we have

$$\sum_{k=1}^{\infty} \frac{|\lambda_k(A)|^{p_k}}{p_k v_A^{p_k}} \leqslant \gamma_\pi(A/v_A),$$

where $v_A = \|A\|_H$ if $\|A\|_H > 1$ and $v_A = 1$ if $\|A\|_H \leqslant 1$.

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