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SIMILARITY TO AN ISOMETRY OF COMPOSITION OPERATORS ON THE HALF–PLANE

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Abstract. Necessary and sufficient conditions are already known in the Hardy spaces of both the disc and the half plane for a composition operator to be an isometry, by Nordgren in the disc [6] and by Chalendar and Partington in the half plane [2]. All the same, conditions for such an operator to be similar to an isometry have taken much longer to find. We present some necessary conditions for general weighted composition operators to be similar to an isometry, and use them to produce a complete characterisation of the rational composition operators on $H^p(\mathbb{C}^+)$ which have this property.

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